CMPT 225: Data Structures & Programming – Unit 26 – Merge Sort

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Today’s Topics

- The Need for Sorting
- Divide and Conquer
- Merging in Arrays and Lists
- Analysis
- Merge-Sort in Java

Welcome to Sorting Algorithms

- A major topic in Computer Science, both in the theory and in programming practice.
- These cover any process where a data set can be **put into order**, like ordering numbers from smallest to largest, or alphabetical order.
- Many of our data structures take pains to **keep their data sorted**, have a **sorting step**, or depend on **data already being sorted** in order to be efficient.
- Already introduced with **Insertion Sort**, way back in unit 4 on arrays.

Remember This Guy?

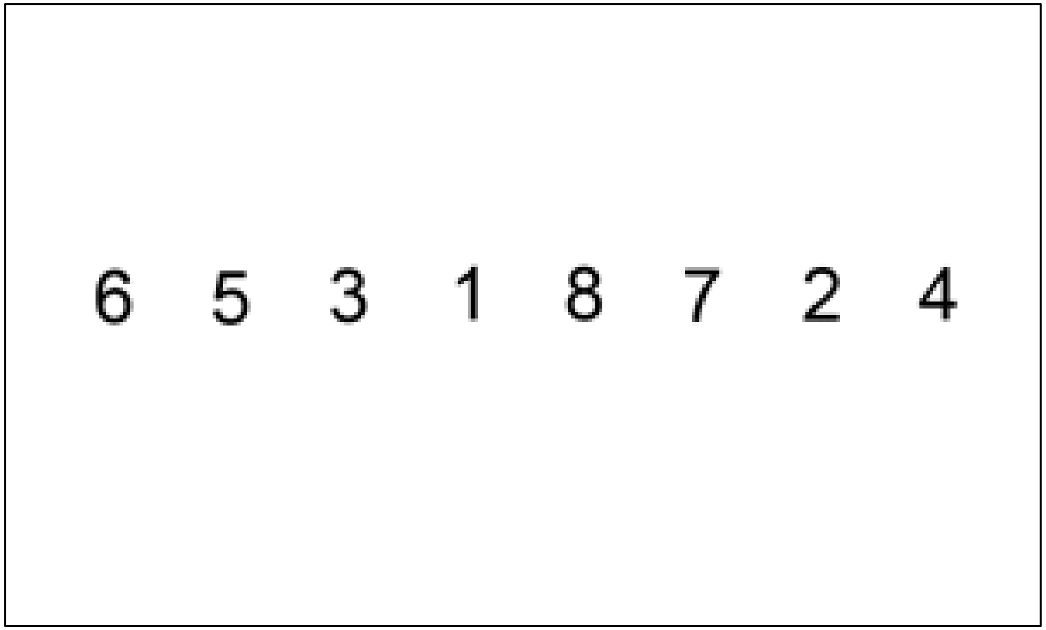


Image credit: https://upload.wikimedia.org/wikipedia/commons/0/0f/Insertion-sort-example-300px.gif

Why Do We Need More Than One?

- **Different sorting methods have different properties** that make them better fits for different circumstances.
- For example, some are only efficient when **paired with certain data structures**, or may **perform more consistently** even if they share a similar O().
- We’ll perform an **initial analysis** of each algorithm as they’re introduced, and then **again once all our sorting options are available**.

Divide-and-Conquer Algorithms

- A common **algorithmic design pattern** we can apply to sorting is **Divide-and-Conquer**.
- We **divide the problem** to be solved into multiple smaller problems that are easier to solve before **combining** them back together again.

The Three Steps of a Divide-and-Conquer Algorithm

1. **Divide**: If the input size is smaller than a certain threshold (say, one or two elements), solve the problem directly using a straightforward method and return the solution. Otherwise, divide the input data into two or more disjoint subsets.
2. **Recur**: Recursively solve the subproblems associated with the subsets.
3. **Conquer**: Take the solutions to the subproblems and “merge” them into a solution to the original problem.

Sorting Through Divide-and-Conquer

- Sorting a long sequence of values can be imagined as answering many smaller questions **comparing pairs of values** to decide which belongs in front of which.
- **Merge-Sort** will let **us break our sequence down** to a set of those smaller comparison problems, solve those, and then **Merge our smaller sorted sequences** back together again.

Merge-Sort as Divide-and-Conquer

1. **Divide**: If S has zero or one element, return S immediately; it is already sorted. Otherwise (S has at least two elements), remove all the elements from S and put them into two sequences, S1 and S2, each containing about half of the elements of S; that is, S1 contains the first n/2 elements of S, and S2 contains the remaining n/2 elements.
2. **Recur**: Recursively sort sequences S1 and S2.
3. **Conquer**: Put back the elements into S by merging the sorted sequences S1 and S2 into a sorted sequence.

Let’s Get One Of Those Wikipedia Visualizations In Here

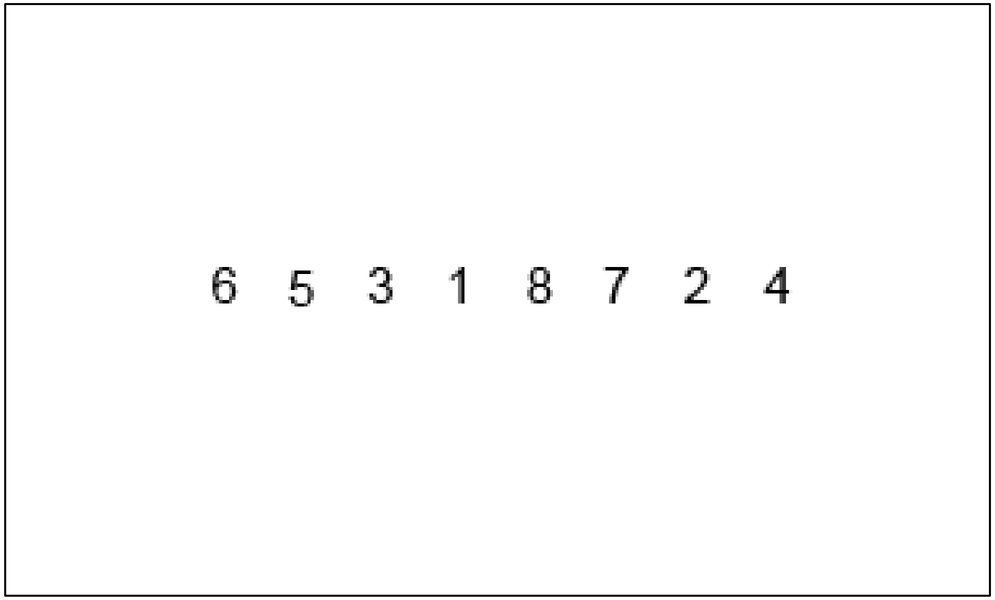
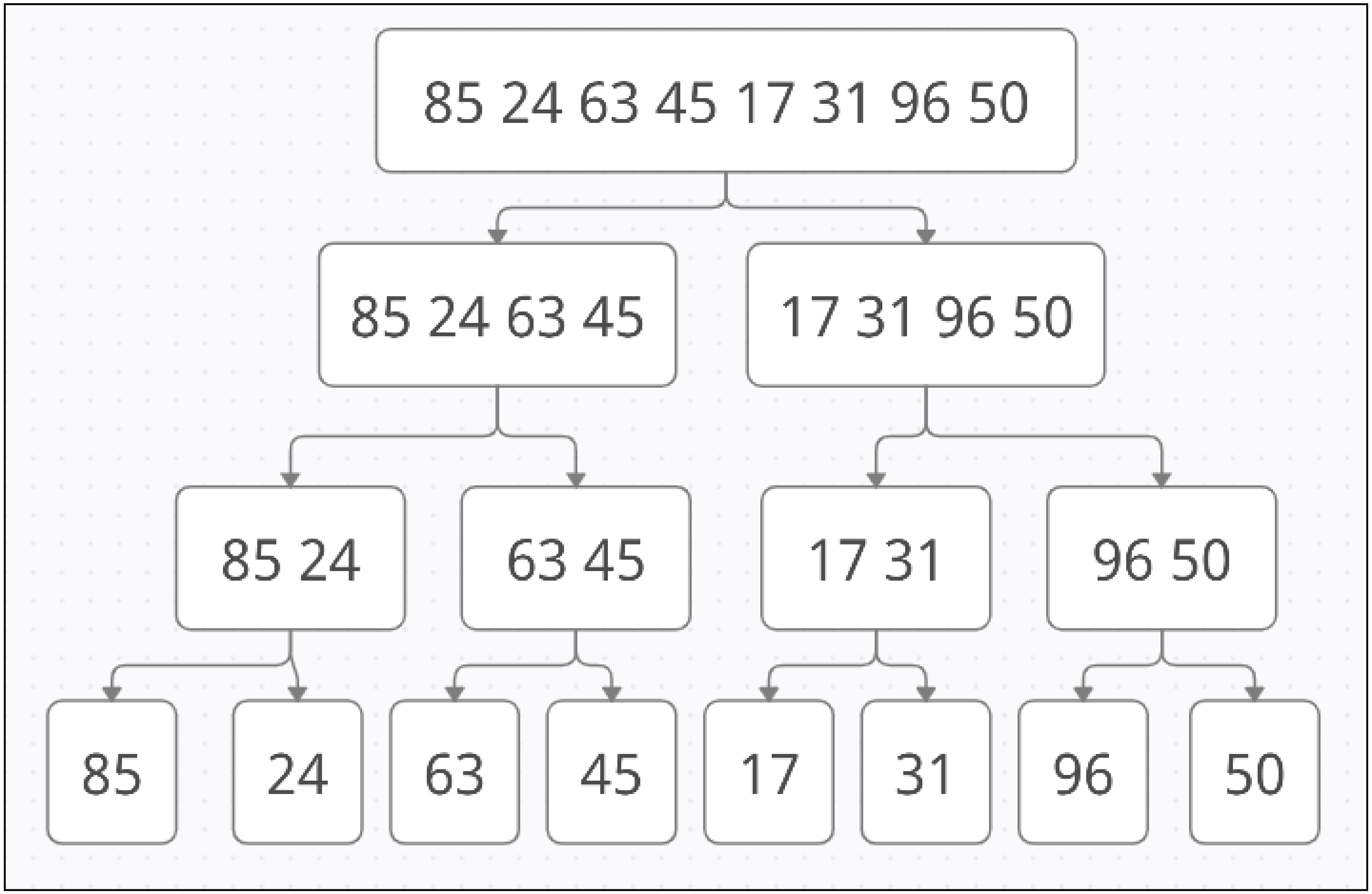


Image credit: https://en.wikipedia.org/wiki/File:Merge-sort-example-300px.gif

Divide…


85 24 63 45 17 31 96 50
85 24 63 45
17 31 96 50
85 24
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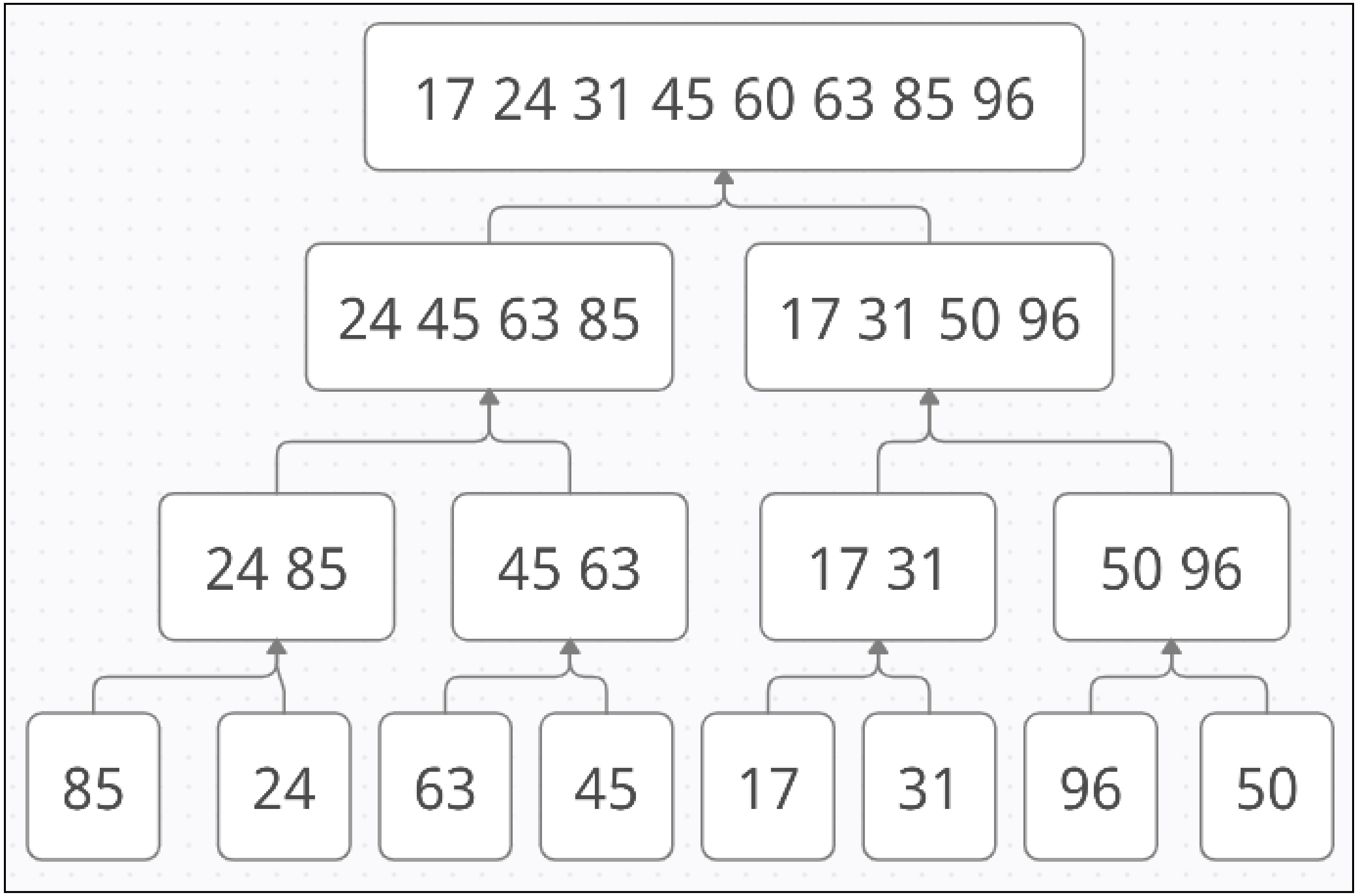
…and Conquer



17 24 31 45 60 63 85 96

24 45 63 85

17 31 50 96

24 85

45 63

17 31

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85

24

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17

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96

50

Ahahaha It’s A Tree Again

- Surprise!
- **Merge-Sort** is performing **Binary Recursion** to implicitly create a **Complete Binary Tree**.
- This is also a **Decision Tree**, because each node of the Tree is essentially a question about which value belongs before which.
- All **Comparison-Based Sorting Algorithms** produce a Decision Tree, and Decision Trees have some common properties (like their height) which will come into play later.

Merging in Arrays and Lists

- The **Merge step** is the most involved to implement, since it depends more on our choice of underlying data structure.
- As such, there are **two slightly different algorithms for merging** depending on whether we're using an **Array** or a **List**, but they're ultimately doing the same thing (combining two smaller sorted sequences into one larger sorted sequence).

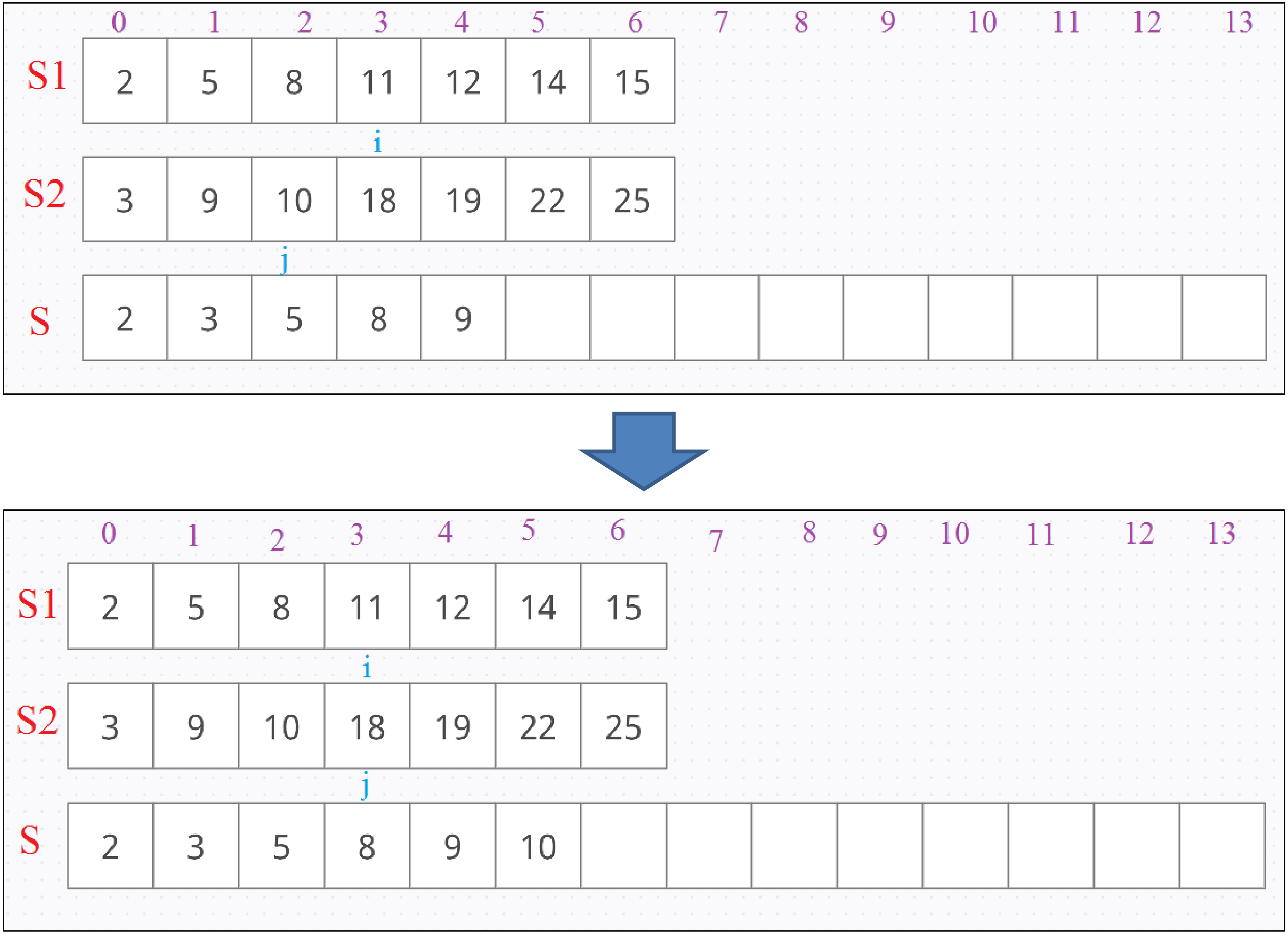
Merge-Sort Algorithm for Arrays

Algorithm merge(S1,S2,S):

Input: Sorted sequences S1 and S2 and an empty sequence S, all of which are implemented as arrays.

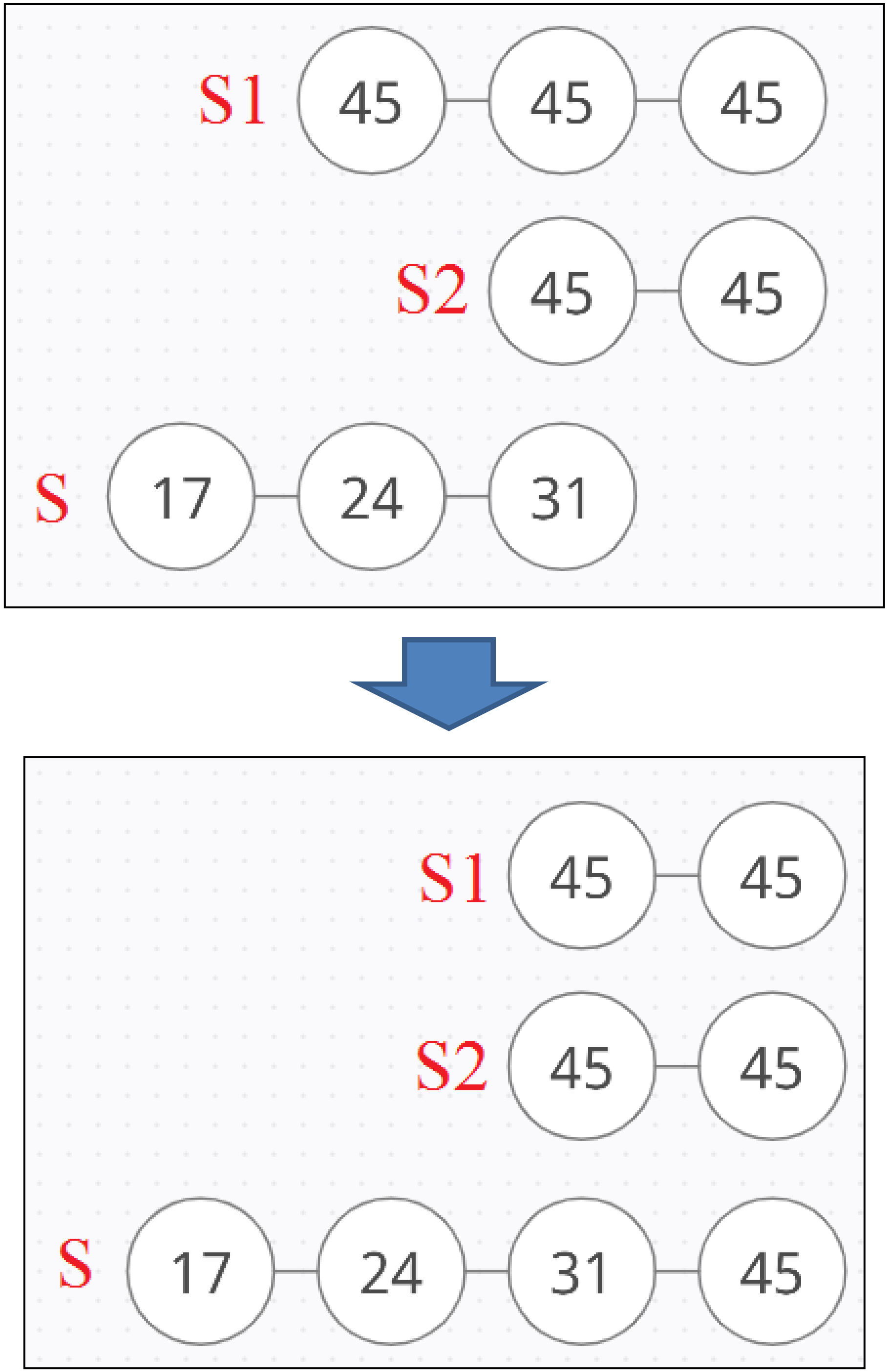
Output: Sorted sequence S containing the elements from S1 and S2.

```
i <- j <- 0
While i < S1.size() and j < s2.size() do
    if S1.get(i) <= S2.get(j) then
        S.addLast(s1.get(i))
        i <- i+1
    else
        S.addLast(S2.get(j))
        j <- j+1
While i < S1.size() do
    S.addLast(S1.get(i))
    i <- i+1
While j< S2.size() do
    S.addLast(S2.get(j))
    j <- j+1
```


0 1 2 3 4 5 6 7 8 9 10 11 12 13
S1
2 5 8 11 12 14 15
i
S2
3 9 10 18 19 22 25
j
S
2 3 5 8 9
0 1 2 3 4 5 6 7 8 9 10 11 12 13
S1
2 5 8 11 12 14 15
i
S2
3 9 10 18 19 22 25
j
S
2 3 5 8 9 10

Merge-Sort Algorithm for Lists

```
Algorithm merge(S1,S2,S):
Input: Sorted sequences S1 and S2 and an empty sequence S,
implemented as linked lists.
Output: Sorted sequence S containing the elements from S1 and S2.
While S1 is not empty and S2 is not empty do
        if S1.first().element() <= S2.first().element() then
                S.addLast(S1.remove(S1.first()))
        else
                S.addLast(S2.remove(S2.first()))
While S1 is not empty do
        S.addLast(S1.remove(S1.first())).
While S2 is not empty do
        S.addLast(S2.remove(S2.first()))
```


S1
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S2
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45
S
17
24
31
S1
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S2
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S
17
24
31
45

Analyzing Merge-Sort

- Each **layer** of the Merge-Sort Decision Tree requires **O(n) comparisons** to resolve, since each layer has n data values in it and every value may need to be compared to find its place in the next round of sequences.
- The **height** of the Tree is **log n**, since it’s a **Complete Binary Tree**.
- With log n layers to resolve at O(n) per layer **Merge-Sort is O(n log n**).
- (Small assumption: this **assumes that comparing two values takes O(1)**, which may not be true of all data, like a complex key.)

Merge-Sort in Java

- Does Java offer built-in Merge-Sort? **No**.
- Well, not directly. Not all the time.
- The **.sort()** function found **in java.util.Collections for Java 7** was a Merge Sort, but was **replaced in Java 8**.
- Sorting in particular is a topic where Java doesn't care to implement every possibility, and will even change the underlying algorithm for common functions based on the current trend or newly developed variants.
- Don't worry, though, I'm sure you can implement Merge-Sort on your own!

Recap – Merging the Key Points Into One Slide

- We introduced the topic of **Sorting Algorithms** and our motivation for exploring them.
- **Merge-Sort** is the first (after **Insertion-Sort**) of our sorting methods, which uses **Divide-and-Conquer** to produce a **Decision Tree**.
- The **Merge step** does most of the actual work, and looks different depending on whether our data structure is **array- or list-based**.
- Merge-Sort runs **in O(n log n)**.
- **Java does not offer Merge-Sort** by default, though it does offer sorting.