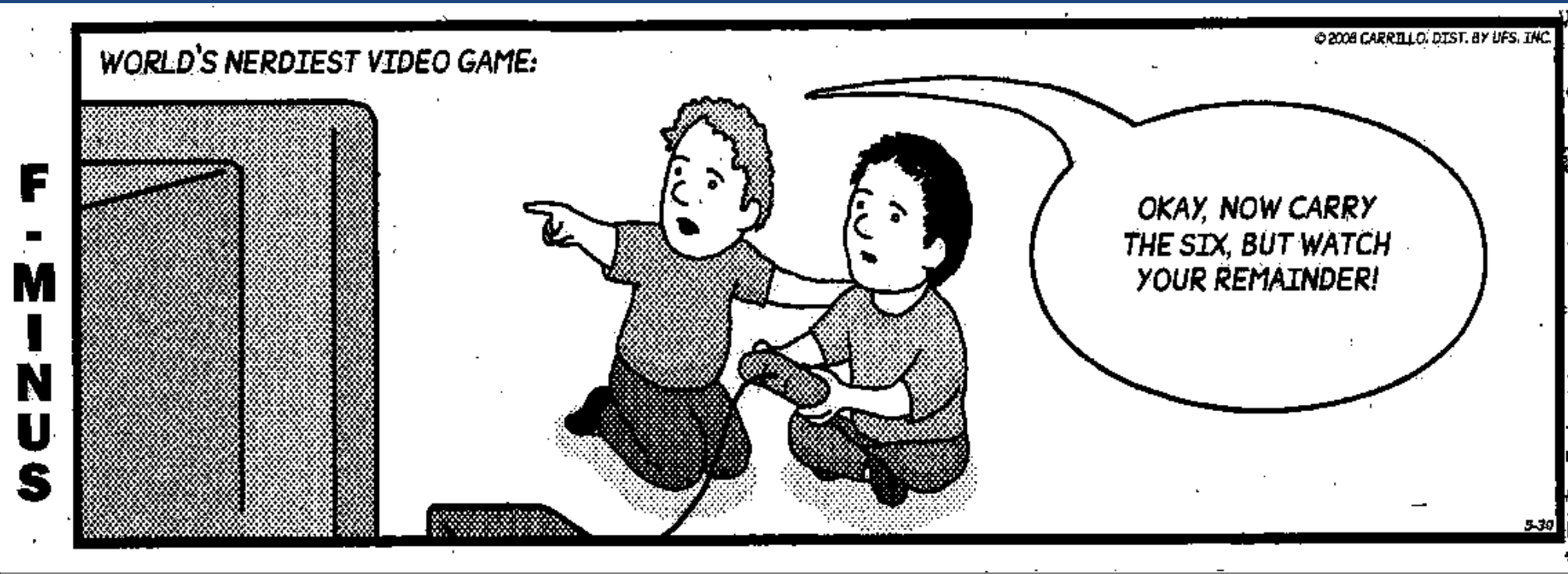
Binary & Hex Representation



How can we store integer values?

Data Types

Data Types

- Humans understand the “type” of data from the meaning of the words:
  - “Hi” - word (i.e., a string)
  - 18,537 - number
- A computer stores all information as 1's and 0's
  - Both “Hi” and 18,537 are stored as:
    = 0100 1000 0110 1001 (which is..)
    = 0x4869
  - Computer must know the type of information!

Dear *human*:
Do you want me to say “Hi”, or print the number 18,537?!?

Data Types

- There are a few different types of data:
  -
    - Integers: Whole numbers like 0, -14, 8382.
    - Floating point: Fractional values like -1.1, 3.14
  -
    - Character: A single character like 'h', 'i', '!'
    - String: A sequence of characters like "Hello!"

Understanding Bits, Bytes

- Bit: a single 0 or 1 in binary.
- Given N bits, there are..
  - 3 bits gives $2^3 = 8$ unique values.
- Byte: 8 bits.
  - How many unique values?

Bit 1	Bit 2	Bit 3
0	0	0
0	0	1
0	1	0
0	1	1
1	0	0
1	0	1
1	1	0
1	1	1

8 possible values with 3 bits.

Large Sizes

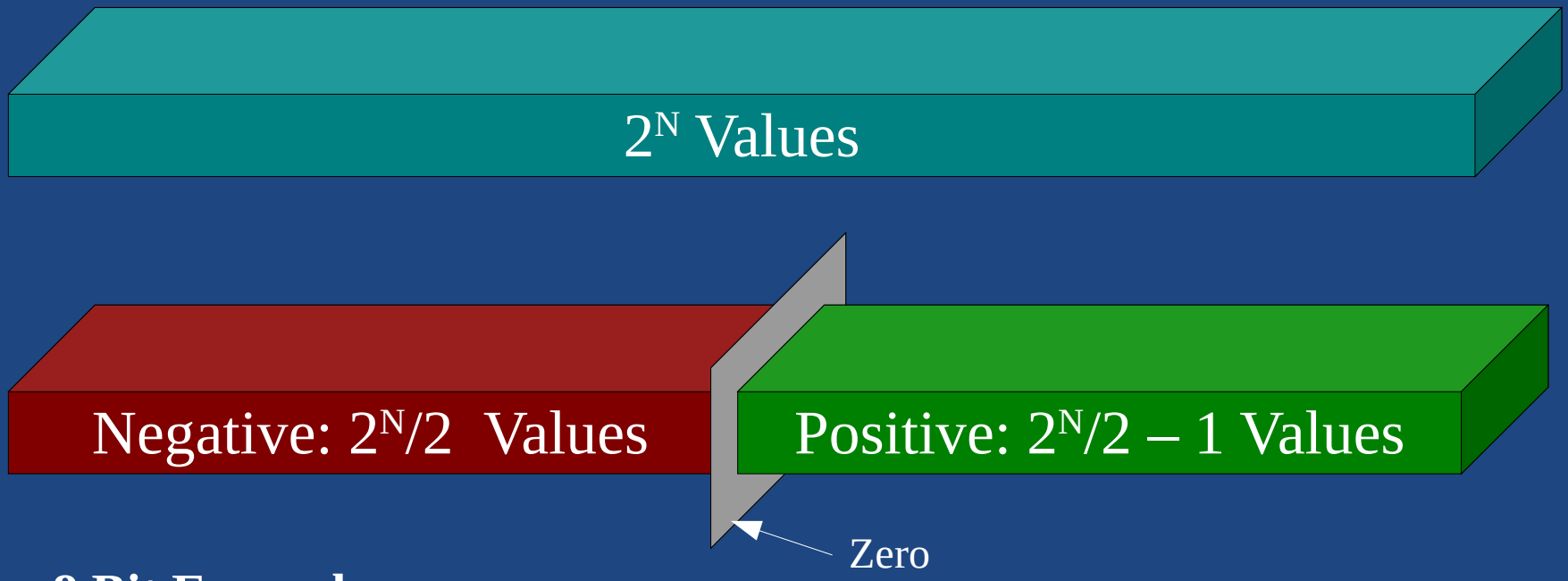
- We often work with large collections of bytes.

Abbr	Name	Approx. Size	Exact Size	*Example* of what you can store
b	Bit		1 bit	Just a 0 or a 1.
B	Byte		8 bits	A character ('a', 'z', '!'..)
	..		~4 bytes = ..	Count of number of people in Canada
kB	..	Thousand bytes	2^{10} bytes = 1024	Your resume.
MB	..	Million bytes	2^{20} bytes	An MP3.
GB	..	Billion bytes	2^{30} bytes	A movie.
TB	..	Trillion bytes	2^{40} bytes	All your pictures, MP3s, and some movies.

More correctly, “kibibyte” is actually 2^{10}, but usually called kilobyte.

Signed Numbers

- Given 2^N different values, how can you represent positive and negative numbers?



8 Bit Example:

- Unique Values: $2^8 = 256$
- Can Express: -128 to +127

Integer Data Types

Integers

Typical Sizes & Range

Type	# Bits	Range
signed char	8 bits	-128 to 127
short	16 bits	-32,768 to +32,767
int	32 bits	~ +/- 2 billion
long	32 bits	~ +/- 2 billion
long long	64 bits	~ +/- 9 quintillion 9,223,372,036,854,775,807

(C++ also has unsigned version of each which only do +'ve values. For example, unsigned long is 32 bits with range 0 to ~4 billion = 2^{32})

- Size (# bits) of each value...
  - int could be 16 bits or 32 bits (or something else!)

The size of data types

- The C++ standard does not specify exactly how big each data type must be.
  - You can..
    (gives size in bytes)

```
#include <iostream>
using namespace std;
int main()
{
    int height = 6;
    cout << "Size of char:\t" << sizeof(char) << endl;
    cout << "Size of height:\t" << sizeof(height) << endl;
    cout << "Size of int:\t" << sizeof(int) << endl;
}
```

Output:

```
Size of char:       1
Size of height:     4
Size of int:        4
```

sizeof() accepts either
the type (like int or float)
or a variable (like height)

Part 2

How does the computer store numbers?

Binary Representation

Storing values

- Computers operate in binary:
  All values are..
  - Digital hardware represents on/off.
  - Use this to store numbers
  - Interpret numbers as text, machine language, music, images, etc.

Base 10

- In base 10, the “place value” is **10** such that the next digits to the left increases in value 10 times
  - Count: 1, 2, 3, ... 9, 10, 11, .
- (Trivial) Example
  Express 2513_{10} in base 10.

Place Value

Digits

Value $= 2000 + 500 + 10 + 3$

$= 2513_{10}$

Hexadecimal: Base 16

- Hexadecimal: base 16, “place value” multiplied by 16
  - Count: 1, 2, 3, ... 9...
- Writing Notation: “0x” prefix shows value is hex
  - int favNum = 0x2a;
- Example: Express 0x2b1a in base 10.

Place Value				
Digits	*2	*b	*1	*a
	(2)	(11)	(1)	(10)
Value	= 2*4096	+ 11*256	+ 1*16	+ 10*1
	= 11034_{10}			

Binary: Base 2

- In base 2, the “place value” is worth 2
  - Count:..
  - In C++ version `14, you can write 0b10111001
    (Need compiler option -std=c++14)
- Example:
  Express 1011 1001_2 in base 10.

Place Value								
Digits	*1	*0	*1	*1	*1	*0	*0	*1
Value	= 128		+ 32	+ 16	+ 8			+ 1
	= 185							

Review

- Express 0x121 in decimal.

- Express $0110\ 1011_2$ in decimal

Decimal to Binary

Part 3

- To convert decimal to binary (or hex),..
- Example: Convert 94 to 8-bit binary.

	(128)	(64)	(32)	(16)	(8)	(4)	(2)	(1)
Place Value	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
Digits			*0		*1: 14 - 8 = 6	*1: 6 - 4 = 2	*1: 2 - 2 = 0	*0
Answer			..					

Between Hex and Binary

- Each Hex digit directly..
- Examples: Hex to Binary
  - 0x9F =..
  - 0xDEAD C0DE =
- Example Binary to Hex
  - $1010\ 0011\ 0001\ 0101_2$ =..

Hex	Binary
0x0	0000
0x1	0001
0x2	0010
0x3	0011
0x4	0100
0x5	0101
0x6	0110
0x7	0111
0x8	1000
0x9	1001
0xA	1010
0xB	1011
0xC	1100
0xD	1101
0xE	1110
0xF	1111

Review

- Convert 50_{10} to binary.
- Express 0xF28C in binary.
- Express $1011\ 1010\ 0100\ 0000_2$ in hex.

Binary Arithmetic and Signed Numbers

Addition

- Adding binary just like adding base 10, but..
- Example:

```
    101011
+   001011
=
```

- Overflow:

```
+   001010
=
```

Negative Numbers

- How can we make a number negative if we don't have the + or – sign? (just 0's and 1's!)
- Idea 1:
  Make the first bit represent + (1) or – (0)

  Limitations:
  - ..
  - Special hardware to maintain the sign bit.

Radix Complement

- Motivation: Addition is easier..
  - Easier for hardware to support only addition.
- Complement
  - Complement of X, in base b, with n digits is:..
- Base 10 Example
  - Complement of 24, base 10, 2 digits:
  - Complement of 24, base 10, 3 digits:

Subtraction

- Replace subtraction with addition:
  Replace A-B with:..
- Example (Base 10, 2 digits):
  - 55 – 32
    = 55 + complement(32)
    = 55 + (100 – 32)
    = 55 + 68
    = 123
  - Why didn't that work?..
- Change subtraction to finding complement, but that..

2's Complement

- Easy way to find 2's complement (base 2):
  1)..
  2)..
- Find the 2's complement of:

	5 (4 bits)	31 (8 bits)	127 (8 bits)
Binary			
Inverted Bits			
+1 (Complement)			

- Interpretation of bits:..

$$= -2^n \; 2^{n-1} \; \ldots \; 2^2 \; 2^1 \; 2^0$$

Negative Numbers

- Computers often use 2's complement notation to represent..
  - Positive numbers written as..
  - Negative numbers written as..
- Possible tasks
  - "Find the 2's complement of X", means..
  - "Express X in 2's complement notation", means
    - if X is positive, just..
    - if X is negative, convert abs(X) to binary, do 2's complement to get negative.

Review

- What is the sum of the following (in 16-bit binary)?

```
  1101 0101 1010 1010
+ 0101 0110 1100 1001
---------------------
```

- What is the 2's complement (4 bits) of 3?

- Express these in 8-bit 2's complement notation:

15 -86

Addition and Subtraction

- Solve the following in 8 bit 2's complement

Complement of 31 (0001 1111)

5 + 5	5 + 5	71 - 31	127 + 6
	0000 0101	0100 0111	0111 1111
+	0000 0101	1110 0001	0000 0110
=			

- Overflow for ans=X+Y:
  If X & Y same sign, and ans is opposite sign.

Meaning of Binary Values

- The meaning of bits depends on the representation.
- What is the value of the following as..

Unsigned	Binary Value	Signed
0	000	0
1	001	1
2	010	2
3	011	3
4	100	-4
5	101	-3
6	110	-2
7	111	-1

Exercise: 2's Complement Subtraction

- Solve (6 – 2) using 2's complement (4 bit):

- Solve (-3 – 2) using 2's complement (4 bit):

  - Overflow for ans=X+Y:
    If X & Y same sign, and ans is opposite sign.

Summary

- Understand bits & bytes of variable types.
  - char, short, int, long, long long
- Conversions between base 10, 16, and 2.
  - Easy conversion between hex and binary.
- Negative numbers and 2's complement.
  - Expressing signed numbers.
  - Subtraction using addition.